Universal spacetimes and modified gravities

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Universal:

S. Hervik, V. P., A. Pravdová, *Type III and N universal spacetimes*, CQG **31**, 215005 (2014)

S. Hervik, T. Málek, V. P., A. Pravdová, *Type II universal spacetimes*, CQG **32**, 245012 (2015)

S. Hervik, V. P., A. Pravdová, *Universal spacetimes in four dimensions*, JHEP 1710: 28 (2017)

M. Kuchynka, T. Málek, V. P., A. Pravdová, *Almost universal spacetimes in higher-order gravity theories*, Phys. Rev. D 99, 024043 (2019)

VSI and math formalism - older works with:

A. Coley and R. Milson (Halifax, Canada),

H. Reall and M. Durkee (Cambridge, UK)

Why modified theories of gravity?

- Currently, general relativity is the standard model for understanding gravity. It passed most of the experimental tests with flying colors. However, most of the current tests probe weak-field, slow-motion limit.
- Strong-field tests are just starting to emerge. These involve the detection of gravitational waves from black hole mergers and studies of gravity near black holes and neutron stars.
- Open questions on the largest scales:
  Accelerating expansion of the universe (dark energy?),
  galaxy rotation curves (dark matter?)
- Dark matter and dark energy can be incorporated within the framework of GR leading to the so-called ΛCDM model, the standard model of current cosmology.

- The ΛCDM model has provided a good description of the structure of anisotropies of the cosmic microwave background, the formation of large structures, and the accelerating expansion of the universe, but various discrepancies have appeared on the sub-galaxy scales (e.g., "small scale crisis")
- The microscopic nature of dark matter remains unknown (so far attempts for detecting dark matter particles unsuccessful)
- Dark energy density disagrees by many orders of magnitude with vacuum energy theoretical predictions.
- It is thus possible that some discrepancies between GR and observations may involve new physics of a gravitational character.
- Further motivation for modifying gravity, this time on small scales comes from attempts to quantize gravity (e.g. string theory).

Modified theories of gravity

Einstein-Hilbert action

$$S = \int \mathrm{d}^n x \sqrt{-g} \left(\frac{1}{\kappa} (R - 2\Lambda) + \mathcal{L}_{\mathrm{matter}} \right)$$

$$\Downarrow$$

Einstein's equations (G=c=1)

curvature $\quad R_{ab} - \frac{1}{2} R g_{ab} + \Lambda g_{ab} = 8\pi T_{ab} \quad$ matter sources

Einstein spacetimes (vacuum spacetimes)

$$R_{ab} = \frac{2}{n-2} \Lambda g_{ab}$$

Example: quadratic gravity

quadratic gravity

$$S = \int \mathrm{d}^n x \sqrt{-g} \left(\frac{1}{\kappa} (R - 2\Lambda) + \alpha R^2 + \beta R_{ab}^2 + \gamma \left(R_{abcd}^2 - 4 R_{ab}^2 + R^2 \right) \right)$$

$\Rightarrow$ quadratic gravity field equations [Gullu, Tekin, Phys. Rev. D, 2009]

$$\frac{1}{\kappa} \left(R_{ab} - \frac{1}{2} R g_{ab} + \Lambda_0 g_{ab} \right) + 2\alpha R \left(R_{ab} - \frac{1}{4} R g_{ab} \right) + (2\alpha + \beta) \left(g_{ab} \nabla^c \nabla_c - \nabla_a \nabla_b \right) R$$
$$+ 2\gamma \left(R R_{ab} - 2 R_{acbd} R^{cd} + R_{acde} R_b{}^{cde} - 2 R_{ac} R_b{}^c - \frac{1}{4} g_{ab} \left(R_{cdef}^2 - 4 R_{cd}^2 + R^2 \right) \right)$$
$$+ \beta \nabla^c \nabla_c \left(R_{ab} - \frac{1}{2} R g_{ab} \right) + 2\beta \left(R_{acbd} - \frac{1}{4} g_{ab} R_{cd} \right) R^{cd} = 0.$$

- Weyl tensor also appears in the field equations
- 2nd derivatives of Ricci $\Rightarrow$ 4th order equations

Universal spacetimes - definition

Definition

A metric is called k-**universal** if all conserved symmetric rank-2 tensors constructed from the metric, the Riemann tensor and its covariant derivatives up to the k^{th} order are multiples of the metric. If a metric is k-universal for all k then it is called **universal**.

$$T^{[ab]} = 0, \; T^{ab}{}_{;b} = 0 \;\Rightarrow\; T_{ab} = \lambda g_{ab}.$$

Universal spacetimes are vacuum solutions to **all** theories with the Lagrangian of the form

$$L = L(g_{ab}, R_{abcd}, \nabla_{a_1} R_{bcde}, \dots, \nabla_{a_1 \dots a_p} R_{bcde})$$

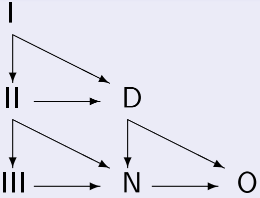
with L being a polynomial curvature invariant. First examples of universal spacetimes discussed in the context of string theory in [Amati, Klimčík, Phys. Lett. B, 1989], [Horowitz, Steif, Phys. Rev. Lett. 1990], and as spacetimes with vanishing quantum corrections in [Coley, Gibbons, Hervik, Pope, CQG. 2008].

Algebraic classification of tensors

Already Einstein's equations are too involved $\Rightarrow$ simplifying assumptions:

- **symmetries** - this is how the Schwarzschild black hole has been discovered in 1915.
- **algebraic type of the Weyl tensor** - Petrov types - this is how the Kerr black hole has been discovered in 1963.

Penrose diagram - algebraic types of the Weyl tensor in 4d



We will need a generalization of the Petrov classification of the Weyl tensor in four dimensions to any tensor in arbitrary dimension.

Main results for Weyl types N and III

Weyl type N

proven in [Hervik, V. P., Pravdová, 2014],

sufficient part discussed without a proof already in [Coley, Gibbons, Hervik, Pope 2008]:

Proposition: Necessary and sufficient condition for type N universal

A type N spacetime is universal if and only if it is an Einstein Kundt spacetime.

Weyl type III

[Kuchynka, Málek, V. P., Pravdová, 2019], partial results in [Hervik, V. P., Pravdová, 2014 and 2017]:

Proposition: Necessary and sufficient condition for Kundt type III universal

Type III Einstein Kundt spacetime is universal if and only if
$F_0 \equiv C_{acde} C_b{}^{cde} = 0, \quad F_2 \equiv C^{pqrs}{}_{;a} C_{pqrs;b} = 0.$

In particular $\tau_i = 0 \Rightarrow F_2 = 0$ and in four dimensions F_0 vanishes identically and thus

Corollary

In 4d Type III, $\tau_i = 0$ Einstein Kundt spacetimes are universal.

Kundt: $\ell_{a;b} = L_{11} \ell_a \ell_b + \tau_i (\ell_a m_b^{(i)} + m_a^{(i)} \ell_b)$
$\tau_i = 0$: ℓ recurrent null vector field

Explicit examples for type II (including D)

Proposition 4 - type D universal spacetimes

$M = M_0 \times M_1 \times \cdots \times M_{N-1}$ (M_0 is Lorenzian)
M_α, $\alpha = 0 \ldots N-1$ are maximally symmetric spaces of dimension n_α and the Ricci scalar R_α

- M is Einstein $\Longleftrightarrow$ $\frac{R_\alpha}{n_\alpha} = \frac{R_0}{n_0}$, $\forall\alpha$
- M is universal $\Longleftrightarrow$ $R_\alpha = R_0$, $n_\alpha = n_0$, $\forall\alpha$

Thus in contrast with Einstein spacetimes, in this way, we can construct universal spacetimes only for **composite** number dimensions.

Kundt extensions:

Proposition 5 - type II universal spacetimes

When M_0 is type N or III universal, M is type II universal.

More general universal spacetimes (e.g. generalized Ghanam-Thompson) likely to exist (we have a proof of 2-universality), however, **no example** is known for **prime number dimensions**. In fact, we have proven:

Proposition 6

In five dimensions, type II (and D) universal spacetimes do not exist.

Almost universal spacetimes/TN spacetimes

Definition: TN spacetimes (or equivalently almost universal spacetimes)

Spacetimes, for which there exists a null vector ℓ such that for every symmetric rank-2 tensor E_{ab} (constructed polynomially from a metric, the Riemann tensor and its covariant derivatives of an arbitrary order) there exist a constant λ and a function ϕ such that

$$E_{ab} = \lambda g_{ab} + \phi \ell_a \ell_b \,.$$

i.e. all tensors E_{ab} are of traceless type N - TN.

Definition: TNS spacetimes

A TN spacetime is called TNS if in addition

$$\phi \ell_a \ell_b = \sum_{n=0}^{N} a_n \Box^n S_{ab}$$

where a_i are constants and S_{ab} is traceless Ricci tensor.

Necessary conditions for almost universal spacetimes

Proposition

Almost universal spacetimes are necessarily CSI.

Proposition

Non-Einstein almost universal spacetimes are necessarily CSI Kundt of Weyl type II or more special.